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**Observations**

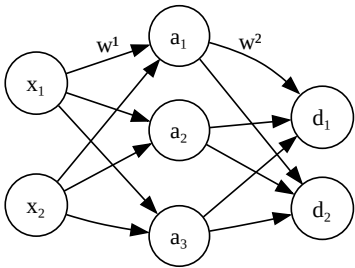
$$i, j, k, l, L, m, M, n, N, o \in \mathcal{N} \tag{1}$$

$$X \in \mathcal{R}^{n \times o} \tag{2}$$

$$Y \in \mathcal{R}^{n \times m} \tag{3}$$

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**Neural Network**



$$a^0 = x_{1 \times p}(n) \tag{4}$$

$$a^L = d_{1 \times m}(n) \tag{5}$$

$$a^l = \varphi(z^l) \tag{6}$$

$$z^l = a^{l-1}W^l \tag{7}$$

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**Gradient Descent**

$$e(n) = y(n) - d(n) \tag{8}$$

$$\xi(n) = \frac{1}{2}ee^\top \tag{9}$$

$$\xi(n) = \frac{1}{2} \sum_{j=1}^M (e_j(n))^2 \tag{10}$$

$$W_{(k+1)} = W_{(k)} - \nabla_W \xi(d, y) \tag{11}$$

$$\xi_{avg}(n) = \frac{1}{2n} \sum_{n=1}^N \sum_{j=1}^M (e_j(n))^2 \tag{12}$$

$$\tag{13}$$

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**Backpropagation**

$$\frac{\partial \xi}{\partial \omega_{ij}^l} = \delta_j^l \frac{\partial z_j^l}{\partial \omega_{ij}^l} \tag{14}$$

$$\delta_j^l = \frac{\partial \xi}{\partial z_j^l} \tag{15}$$

$$\frac{\partial z_j^l}{\partial \omega_{ij}^l} = a_i^{l-1} \tag{16}$$

$$\tag{17}$$

**Output Layer**

$$\delta_j^L = \frac{\partial \xi}{\partial z_j^L} = \frac{\partial \xi}{\partial a_j^L} \frac{\partial a_j^L}{\partial z_j^L} \tag{18}$$

$$\delta_j^L = \frac{\partial \xi}{\partial a_j^L} \dot{\varphi}(z_j^L) \tag{19}$$

$$= -e_j \dot{\varphi}(z_j^L) \tag{20}$$

**Hidden Layer**

$$\delta_j^l = \frac{\partial \xi}{\partial z_j^l} = \sum_k \frac{\partial \xi}{\partial z_k^{l+1}} \frac{\partial z_k^{l+1}}{\partial z_j^l} \tag{21}$$

$$\delta_j^l = \sum_k \delta_k^{l+1} \frac{\partial z_k^{l+1}}{\partial z_j^l} \tag{22}$$

$$\frac{\partial z_k^{l+1}}{\partial z_j^l} = \frac{\partial}{\partial z_j^l} \left( \sum_j \omega_{jk}^{l+1} \varphi(z_j^l) \right) \tag{23}$$

$$\frac{\partial z_k^{l+1}}{\partial z_j^l} = \omega_{jk}^{l+1} \dot{\varphi}(z_j^l) \tag{24}$$

$$\delta_j^l = \sum_k \delta_k^{l+1} \omega_{jk}^{l+1} \dot{\varphi}(z_j^l) \tag{25}$$

$$\tag{26}$$